ANALYSIS OF MEASLES DISEASE IN INDIVIDUAL USING BASIC SIR MODEL

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Abstract In this paper, we examine the measles pandemic using basic SIR model. We investigate endemic equilibrium points, disease-free equilibrium points, reproduction number as well as basic reproduction numbers to examine the effects of various partition transitions. It is proposed that the measles model is both locally and globally asymptotically stable at the disease-free equilibrium point.

Keywords: SIR Model, Stability Analysis, Reproduction Number, Basic Reproductive Number, Euler Method

Introduction

Undoubtedly, the heinous epidemics of infectious illnesses have moulded human history; entire countries and civilizations have been took up the majority of the map over the ages. The Peridean Golden Era began in Athens in 425 BC. The epidemic known as "Cocoliztli" in the 16th century claimed 13 million lives, eradicating the native population of Mesoamerica. The Black Plague, which struck Europe in 1348, is believed to have killed over 25 million people in just five (5) years.

S.J. Liao proposed the new analytical method in 1992 [1].Adolescents who display bodily symptoms are stigmatized by community and get unequal treatment, as noted by Chapman [2]. A general analytical method called the Homotopy Analysis Method is used to solve nonlinear equations, and the results are given as series.A paper on the Invasion, Persistence, and Spread of Infectious Disease among Animal and Plant Communities was proposed by Anderson and May [3].According to Jefferies [4], AIDS is a novel sexually transmitted infection (STD) that develops in HIV-positive individuals.A. I. Enagi [5] examines a probabilistic compartmental framework of the national tuberculosis and leprosy control program's tuberculosis control strategy.

Mugisha et al.'s [6] mathematical models for the dynamics of tuberculosis in densely populated areas were developed in order to reduce and ultimately eliminate tuberculosis.Jama [7], the World Health Organization's most recent Global TB Report 2017 states that although the disease's fatality rate has decreased by 37% globally since 2000, tuberculosis (TB) is still one of the leading ten root causes of death. In order to alleviate the impact of this illness and achieve the 2030 target of putting an end to the TB epidemic, more political will is required.In his work, Lerner [8] revisited Thomas Holnee's (2009–2015) [9] tuberculosis investigation and looked at the relationship between strain and tuberculosis.Theorell [10] has questioned their findings, citing its failure to take into account the reality that each person reacts to challenging circumstances uniquely.Bhavithra.et.al in [11] found the value for R0 and explained its behaviour.

I utilized the SIR Model to observe this topic—the spread of the measles—in order to forecast or calculate the rate at which the work will be completed.

Basic terminologies

Susceptible (S): Individuals who are not infected but can become infected if they come into contact with an infected person.

Infected (I): Individuals who are currently infected and can transmit the disease to susceptible individuals.

Recovered (R): Individuals who have recovered from the disease and are assumed to be immune, thus not susceptible to the disease anymore.

Mathematical model

Let us consider the following mathematical model

$$\frac{dS}{dt} = (1-\mathrm{p})\pi - \beta SI - \pi S$$

$$\frac{dI}{dt} = \beta SI - (\gamma+\pi)I$$

$$\frac{dR}{dt} = p\pi+\gamma I-\pi R$$

Parameter	Description	Values
S	Those who are susceptible	1
I	Those who are infected	0
R	Recovered people	0
μ	Rate of mortality from natural causes	0.5
β	Typical rate of interaction	0.9
γ	Rate of recovery	2.04
π	Fertility rate	0.2
P	yearly vaccinations for newborns	0.9

Extensive Analysis:

We will examine the framework in two categories

(i) Measles free equilibrium point

(ii) Endemic equilibrium point

(i) Measles free equilibrium point:

The concept of the measles-free equilibrium is crucial for understanding the conditions under which a disease can invade a population or, conversely, the conditions necessary to eradicate a disease.

As there is no infection in the population, we have found MFE as $M_0 = (1-p, 0,0)$

Reproduction number and Basic reproduction number:

Basic Reproduction Number (R_0): This is the average number of secondary infections produced by one infected individual in a completely susceptible population. Reproduction Number (R): it reflects the average number of secondary infections in a population that may not be fully susceptible.

$R = \frac{\beta}{\gamma+\pi}$ and $R_0 = \frac{\beta S}{\gamma+\pi}$

Endemic equilibrium point:

The endemic equilibrium is obtained by solving the following equations

The endemic equilibrium point is given by $M_1 = (S_1, I_1, R_1) = (\frac{1-p}{R_0}, \frac{\pi}{\beta}(R_0\text{-}1), \text{p+}\frac{R_0-1}{\beta})$

Stability Analysis:

Stability analysis in mathematical modeling, including epidemiological models like the SIR model, is used to determine the behavior of the system over time and whether its equilibrium points are stable or unstable.

Local Stability Analysis:

The concept of local stability helps us understand how the system reacts to small disturbances or changes near this equilibrium.

The jacobian of our model at measles free equilibrium point is given by

$$J = \begin{bmatrix} -\pi & -\beta(1-p) & 0 \\ 0 & \beta(1-p)-\gamma-\pi & 0 \\ 0 & \gamma & -\gamma-\pi \end{bmatrix}$$

The characteristic equation is $|J-\lambda I| = 0$

$$\lambda^3 + (p\beta+3\pi+\gamma-\beta)\lambda^2 + (\pi^2+2\pi p\beta+2\pi\gamma+2\pi^2-2\pi\beta)\lambda + (p\beta\pi^2+\gamma\pi^2+\pi^3-\beta\pi^2) = 0$$

By Routh-Hurwitz criteria our model is locally asymptotically stable.

Global Stability analysis:

Let us consider the lyapunov function V_1 as V_1(t,S,I,R) = $C_1 I$

$$\frac{dV_1}{dt} = c_1 I'$$

Let $c_1 = \frac{1}{\beta(1-p)-(\gamma+\pi)}$

Thus $\frac{dV_1}{dt} = 0$ iff I = 0

Now put I = 0 in model system of equation

We get S → 1-p and R → 0 as T → ∞

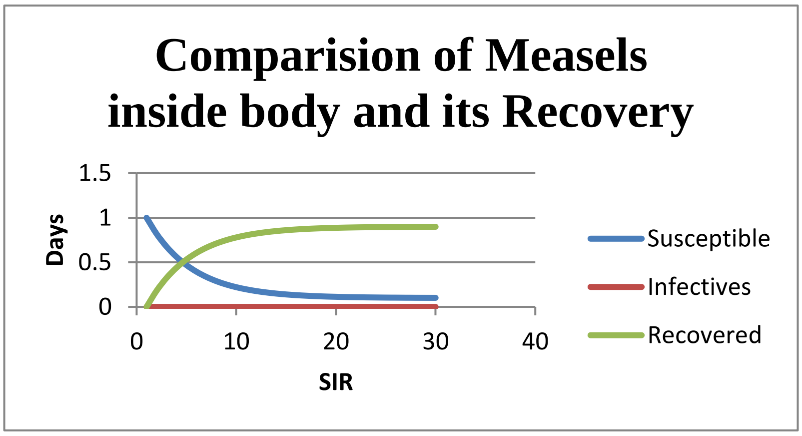
By Lasalle's in-variance principle V_1 is globally asymptotically stable.

Critical vaccination proportion

The formula is: $P_c = 1 - \frac{1}{R} = 1 - \frac{\gamma+\pi}{\beta}$. If $P_c >$ p, measles free equilibrium is locally stable with the coordinates $M_0 = (1-p, 0, 0)$

Analysis of measles:

The initial conditions are S(0) = 1, I(0) = 0 and R(0) = 0



Conclusion:

In this research, we looked at a mathematical model for the measles sickness.For the models in the stability analysis and basic reproduction number are explored.We used the Euler method for numerical simulation and the corresponding results are compared using graphs. Here we have analyzed the recovery of disease inside human body and we found the estimated days when can a person get recovered from the disease.

References

1. Liao S. J. (1998). Homotopy Analysis Method: A New Analytical Method for Nonlinear Problems.

Applied Mathematics and Mechanics, 19(10).

2.Chapman, S. J. (1997). Barley straw decomposition and S immobilization. Soil Biology and Biochemistry, 29(2), 109-114.

3. May, M., Gompels, M., Delpech, V., Porter, K., Post, F., Johnson, M., ... & Hill, T. (2011). Impact of late diagnosis and treatment on life expectancy in people with HIV-1: UK Collaborative HIV Cohort (UK CHIC) Study. Bmj, 343, d6016.

4. Jefferies, J. E., DeMay, R. W., & Lachinyan, G. L.(2014). U.S. Patent No. 8,768,565. Washington, DC: U.S. Patent and Trademark Office.

5. Enagi, A. I. (2013). A deterministic compartmental model of tuberculosis control strategy adopted by the national tuberculosis and leprosy control program in Nigeria. Specific journal of science and technology. 14(1): 342-348.

6. Mugisha, J. Y. T Ssematimba, A., & Luboobi, L. S (2015). Mathematical models for the dynamics of

TB in density – dependent populations; the case of internally displaced peoples' camps (IDPCS) in

Uganda. Biostat, 1(3):217-224.

7. Friedrich, M. J. (2017). Tuberculosis update 2017. Jama, 318(23), 2287-2287.

8. Lerner, M. (2017). The Mind and Faith of Justice Holmes: his speeches, essays, letters, and judicial

opinions. Routledge.

9. Barron, I. H. (2017) can stress cause diseases.Revisiting tuberculosis research oh Thomas

holmes. Ann Intern Med; 124:673-680.

10. Aronsson, G., Theorell, T., Grape, T.,Hammarström, A., Hogstedt, C., Marteinsdottir, I.,& Hall, C. (2017). A systematic review including meta-analysis of work environment and burnout symptoms. BMC public health, 17(1),264.

11. H.A.Bhavithra, S.Sindu devi "Feasibility and Stability Analysis for Basic Measles Model Using Fuzzy Parameter," Contemporary mathematics Vol 5 issue 1 2024,897-912